Lecture Notes, Lecture 27

# Summing Up

Classic Arrow-Debreu general equilibrium model of the economy.

## Economic General Equilibrium

Where did we start?

General Equilibrium Theory: Who was Prof. Debreu and why did he have his own parking space in Berkeley's Central Campus??

Nobel Prizes: Arrow, Debreu

June 1993: A birthday party for mathematical general equilibrium theory!

October 2005: Mathematical Economics: The Legacy of Gerard Debreu http://emlab.berkeley.edu/users/cshannon/debreu/home.htm

June 2009: European Workshop on General Equilibrium Theory,

What does mathematical general equilibrium theory do? Tries to put microeconomics on same basis of logical precision as algebra or geometry. Axiomatic method: allows generalization; clearly distinguishes assumptions from conclusions and clarifies the links between them.

Four ideas about writing an economic theory:

Ockam's razor (KISS - Keep it simple, stupid. ), improves generality

Testable assumptions (logical positivism), avoids vacuity

Link with experience, robustness, Solow "All theory depends on assumptions which are not quite true. That is what makes it theory. The art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive. A "crucial" assumption is one on which the conclusions do depend sensitively, and it is important that crucial assumptions be reasonably realistic. When the results of a theory seem to flow specifically from a special crucial assumption, then if the assumption is dubious, the results are suspect. " (Contribution to the Theory of Economic Growth, 1956)

Precision, reliable results, Hugo Sonnenschein: "In 1954, referring to the first and second theorems of classical welfare economics, Gerard wrote 'The contents of both Theorems ... are old beliefs in economics. Arrow and Debreu have recently treated these questions with techniques permitting proofs.' This statement is precisely correct; once there were beliefs, now there was knowledge.

"But more was at stake. Great scholars change the way that we think about the world, and about what and who we are. The Arrow-Debreu model, as communicated in *Theory of Value* changed basic thinking, and it quickly became the standard model of price theory. It is the 'benchmark' model in Finance, International Trade, Public Finance, Transportation, and even macroeconomics. ... In rather short order it was no longer 'as it is' in Marshall, Hicks, and Samuelson; rather it became 'as it is' in *Theory of Value*." (remarks at the Debreu conference, Berkeley, 2005).

## The Arrow-Debreu Model of Economic General Equilibrium

Brouwer Fixed Point Theorem: Note that BFPT $\Leftrightarrow$ Existence of Walrasian Equilibrium (Uzawa equivalence theorem)

$$P = \left\{ p \middle| p \in R^N,\ p_k \geq 0,\ k = 1\ldots, N,\ \sum_{k=1}^{N} p_k = 1 \right\}$$

$$\tilde{Z}(p) = \sum_{i \in H} \tilde{D}^i(p) - \sum_{j \in F} \tilde{S}^j(p) - r$$

$$= \sum_{i \in H} x^i - \sum_{j \in F} y^j - r$$ , where $x^i$ is household i's consumption plan, $y^j$ is firm j's production plan and r is the resource endowment of the economy. $\tilde{Z}(p)$ is the economy's excess demand function. Recall that all of the expressions in $\tilde{Z}(p)$ are N-dimensional vectors.

**Definition**: $p^0 \in P$ is said to be an equilibrium price vector if $\tilde{Z}(p^0) \leq 0$ (the inequality holds co-ordinatewise) with $p^o_k = 0$ for k such that $\tilde{Z}_k(p^0) < 0$ . That is, $p^0$ is an equilibrium price vector if demand equals supply except for free goods, $\sum_{i \in H} \tilde{D}^i(p^0) \leq \sum_{j \in F} \tilde{S}^j(p^0) + r$ .

Weak Walras' Law (Theorem 6.2): For all $p \in P,\ p \cdot \tilde{Z}(p) \leq 0$. For p such that $p \bullet \tilde{Z}(p) < 0$, there is k = 1, 2, ..., N, so that $\tilde{Z}_k(p) > 0$, assuming C.I - C.V, C.VII, C.VIII.
A consequence of the budget constraint.

Continuity: $\tilde{Z}(p)$ is a continuous function, assuming P.II, P.III, P.V,P.VI and C.I-C.V, C.VII-C.VIII (Theorem 4.1, Theorem 5.2, Theorem 6.1).
See how tightly tied together it is. Supply continuity follows from closedness of tech set, and (partly) from convexity of technology. Income continuity follows from supply continuity. Demand continuity follows from income continuity, continuity of utility, adequacy of income (avoid the Arrow corner), and (partly) from convexity of prefernces.

**Theorem 7.1**: Assume P.II, P.III, P.V, P.VI, and C.I-C.V, CVII-C.VIII. There is $p^* \in P$ so that $p^*$ is an equilibrium.

**Proof:** $T : P \to P$. For each k= 1,2,3, ..., N.

$$T_k(p) \equiv \frac{p_k + \max\left[0, \tilde{Z}_k(p)\right]}{1 + \sum_{n=1}^{N} \max\left[0, \tilde{Z}_n(p)\right]} = \frac{p_k + \max\left[0, \tilde{Z}_k(p)\right]}{\sum_{n=1}^{N} \{p_n + \max\left[0, \tilde{Z}_n(p)\right]\}} .$$

By the Brouwer fixed point theorem there is $p^* \in P$ so that $T(p^*) = p^*$ . But then for all k = 1, ..., N,

$$T_k(p_k^*) = p_k^* = \frac{p_k^* + \max\left[0, \widetilde{Z}_k(p^*)\right]}{1 + \sum_{n=1}^{N} \max\left[0, \widetilde{Z}_n(p^*)\right]}$$

Thus, either $p_k^* = 0$ or

$$p_k^* = \frac{p_k^* + \max[0, \widetilde{Z}_k(p^*)]}{1 + \sum_{n=1}^{N} \max\left[0, \widetilde{Z}_n(p^*)\right]} > 0 \quad .$$

Theorem 7.1 is a proof of the consistency of the competitive model of chapters 4-7. It is possible to find prices, $p^* \in P$ so that competitive markets clear. When economists talk about competitive market prices finding their own level, they are not necessarily speaking vacuously. Under the hypotheses above, there is a competitive equilibrium price system.

Welfare Economics

Separation Theorems (1 lecture)

Fundamental Theorems of Welfare Economics

Fundamental Theorems of Welfare Economics

First Fundamental Theorem of Welfare Economics: Every competitive equilibrium allocation is Pareto efficient.

Does not require convexity, continuity, only monotonicity and unbounded consumption possibility.

Second Fundamental Theorem of Welfare Economics (subject to boundary conditions): Let preferences and technologies be convex. Then for any Pareto efficient allocation ($x^{oi}$, $y^{oj}$), there is $p \in P$, so that the allocation ($x^{oi}$, $y^{oj}$) is a competitive equilibrium at prices p, subject to a redistribution of endowment.

Arrow Possiblity Theorem.

Equilibrium over Time: Futures Markets

Constant Returns, U-Shaped Cost Functions, Concentrated preferences

Three big ideas

Equilibrium: S(p) = D(p)

Decentralization
Efficiency

**Big lesson: Think about economics as a deductive science, reduce analysis to its essentials for generality.**

**Biggest lesson: How to think, Formal general theory**